n this talk we wish to stress the significance and importance of the so-called worm domain in function theory of several complex variables and present some recent results obtained in collaboration with S. Krantz and C. Stoppato.

Starting from the construction of Diederich and Fornæss, we describe the first basic properties of such domain $W_\mu$, which is smooth bounded and pseudoconvex. The boundary of $W_\mu$ is strongly pseudoconvex except on a critical annulus $A$. Here $\mu > 0$ is a parameter that governs many

In particular we illustrate some key properties of $W_\mu$ such as:

- it does not admit a neighborhood of smoothly bounded strongly pseudoconvex domains;
- if $\varrho$ is a defining function for $W_\mu$ and $(-\varrho)^s$ is plurisubharmonic, than $s \leq \nu$, where $\nu = \pi/2\mu$, [DiFo1977];
- the Bergman projection $P$ on $W_\mu$ does not map $H^s(W_\mu)$ to $H^s(W_\mu)$ when $s \geq \nu$, [Ba1992];
- on $W_\mu$ the so-called Condition $R$ of Bell fails, [Ch1996].

We will describe some domains related to $W_\mu$ and the asymptotic expansion of the Bergman kernel and precise mapping properties of the Bergman projection, [KP2008].

In our recent work, in collaboration with S. Krantz and C. Stoppato, we study the Bergman kernel and projection on the unbounded worm

$$W_\infty = \{(z_1, z_2) \in \mathbb{C} \times \mathbb{C}^* : |z_1 - e^{i\log|z_2|^2}| < 1\},$$

where $\mathbb{C}^* = \mathbb{C} \setminus \{0\}$.

We show that the Bergman space of $W_\infty$ is not trivial. In this work we study its Bergman kernel $K$ and projection $P$. We obtain an asymptotic expansion for $K$ that allows us to describe its singularities at the boundary and to prove the following:

1. For all $s > 0$, the Bergman projection $P_\infty$ does not map the Sobolev space $W^s(W_\infty)$ into itself.
2. For $p \neq 2$, $P$ does not map $L^p(W_\infty)$ into itself.

References